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for PID tuning method

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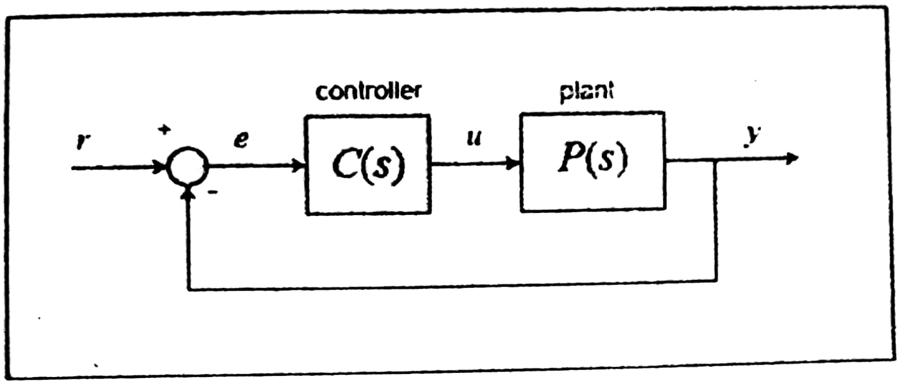
PID Control

In this lecture, we will consider the following unity-feedback system:



PID Control

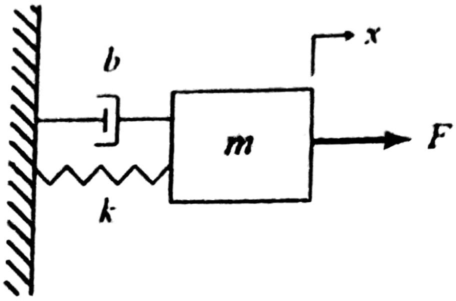
The output of a PID controller, which is equal to the control input to the plant, is calculated in the time domain from the feedback error as follows

$$u(t) = K_p e(t) + K_i \int e(t)dt + K_p \frac{de}{dt}$$

The general effects of each controller parameter (K_p, K_d, K_i) on a closed-loop system are summarized in the table

CL RESPONSE	RISE TIME	OVERSHOOT	SETTLING TIME	S-S ERROR
Kp	Decrease	Increase	Small Change	Decrease
Ki	Decrease	Increase	Increase	Decrease
Kd	Small Change	Decrease	Decrease	

Example Problem



The transfer function I

$$\frac{X(s)}{F(s)} = \frac{1}{ms^2 + bs + k}$$

Example Problem

Let

$m = 1$ kg $\qquad b = 10$ N.s/m $\qquad k = 20$ N/m $\qquad F = 1$ N

Substituting these values into the above transfer function

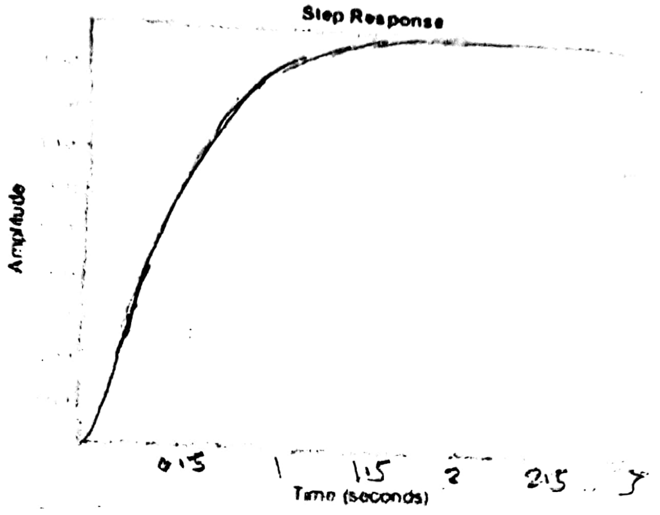
$$\frac{X(s)}{F(s)} = \frac{1}{s^2 + 10s + 20}$$

The closed-loop transfer function of our unity-feedback system with a proportional controller is the following, where $X(s)$ is our output (equals $Y(s)$, and our reference $R(s)$ is the input

$$T(s) = \frac{X(s)}{R(s)} = \frac{K_p}{s^2 + 10s + (20 + K_p)}$$

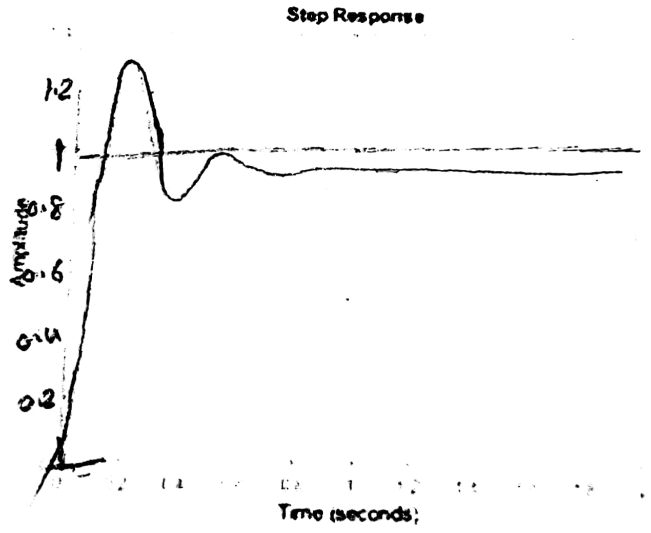
CLOSED LOOP STEP RESPONSE

For kp=1 we get the following closed loop step response



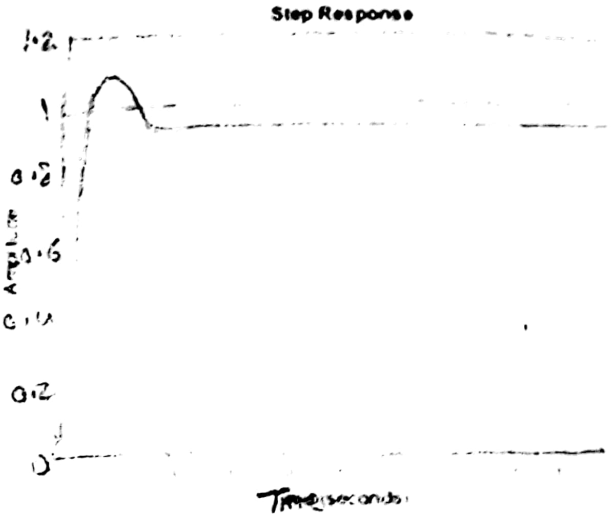
Let the proportional gain (K_p) equal 300

The plot shows that the proportional controller reduced both the rise time and the steady-state error, increased the overshoot, and decreased the settling time by a small amount.



Now let's take a look at PD control. From the table shown above, we see that the addition of derivative control tends to reduce both the overshoot and the settling time. The closed-loop transfer function of the given system with PD controller is

$$T(s) = \frac{X(s)}{R(s)} = \frac{K_d s + K_p}{s^2 + (10 + K_d)s + (20 + K_p)}$$



For Kp=300, Kd=10, we get the side figure

This plot shows that the addition of the derivative term reduced both the overshoot and the settling time, and had a negligible effect on the rise time and the steady-state error

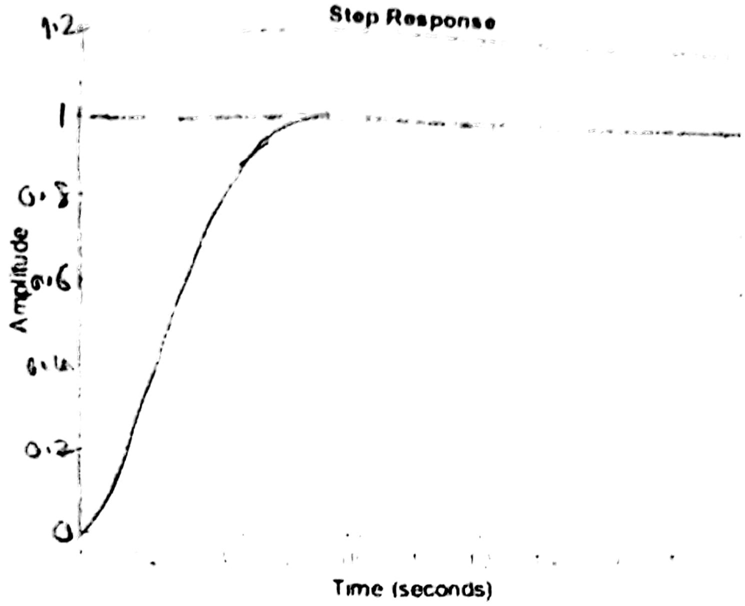
let's investigate PI control. From the table, we see that the addition of integral control

~~(K_I) tends to decrease the rise time, increase both the overshoot and the settling time, and reduces the steady-state error. For the given system, the closed-loop transfer function with a PI controller is~~

k_I tends to increase overshect and decrease rise time and reduce the steady state error. for a given system, the closed loop transfer function is

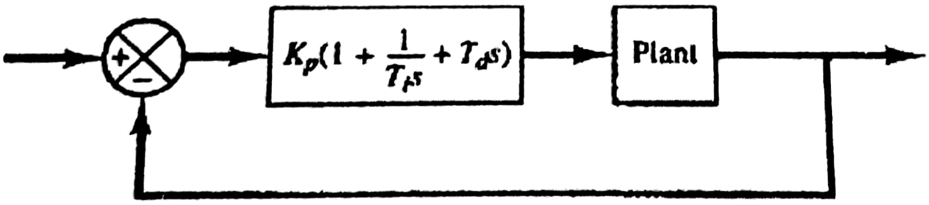
$$T(s) = \frac{X(s)}{R(s)} = \frac{K_p s + K_I}{s^3 + 10s^2 + (20 + K_p)s + K_I}$$

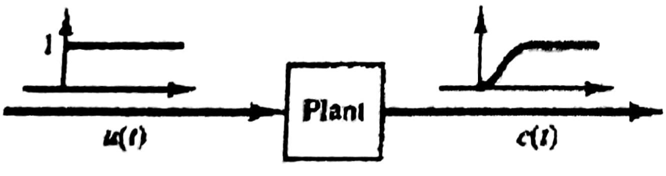
With Kp=30 and Ki=70 the following closed loop response is obtained [we decrease here the Kp because the PI increases the overshoot

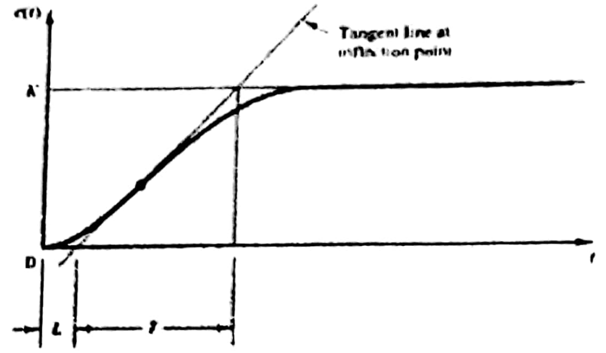


Ziegler–Nichols rules for tuning PID controllers.

First method.







$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$
$$= 1.2\frac{T}{L}\left(1 + \frac{1}{2Ls} + 0.5L\right.$$

Type of Controller	K_p	T_i	T_d
P	$\frac{T}{L}$	∞	0
PI	$0.9\frac{T}{L}$	$\frac{L}{0.3}$	0
PID	$1.2\frac{T}{L}$	$2L$	$0.5L$

Table 1

Second Method.

In the second method, we only first set $T_i = \infty$, $T_d = 0$ and by using the Proportional control action only as shown in figure below [fig. a], increase K_p from 0 to a critical value ($K_p = K_{cr}$) at which the output first exhibits sustained oscillations as shown in Fig.b. (If the output does not exhibit sustained oscillations for whatever value K_p, may take, then this method does not apply.) Thus, the critical gain K_{cr}, and the corresponding period of oscillations P_{cr} are obtained to fill the following design Table (2).

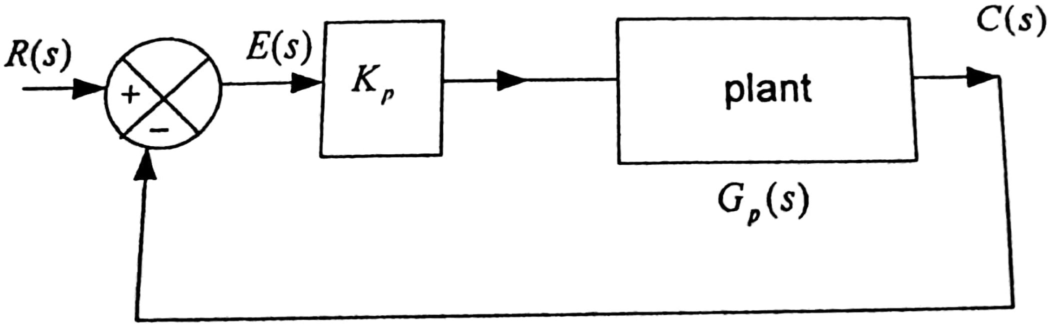


Fig.a

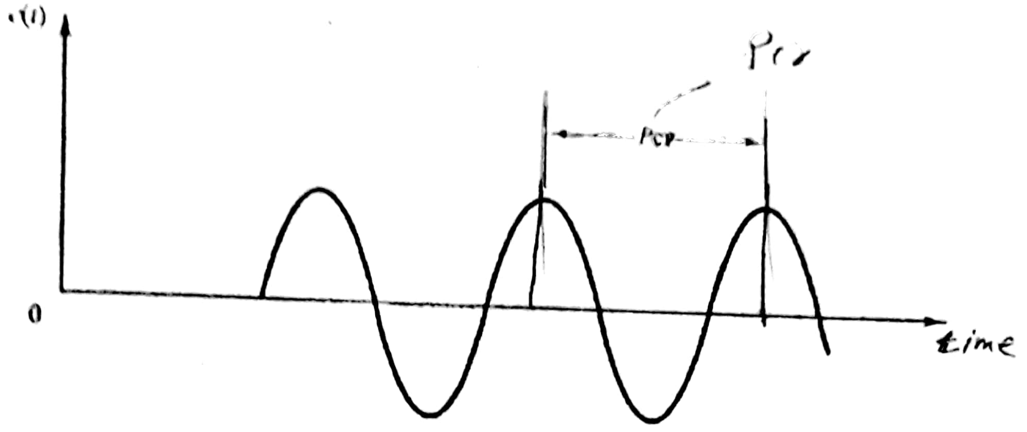


Fig. b

Type of Controller	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

Ziegler–Nichols Tuning Rule Based on Critical Gain K_{cr} and Critical Period P_{cr} (Second Method)

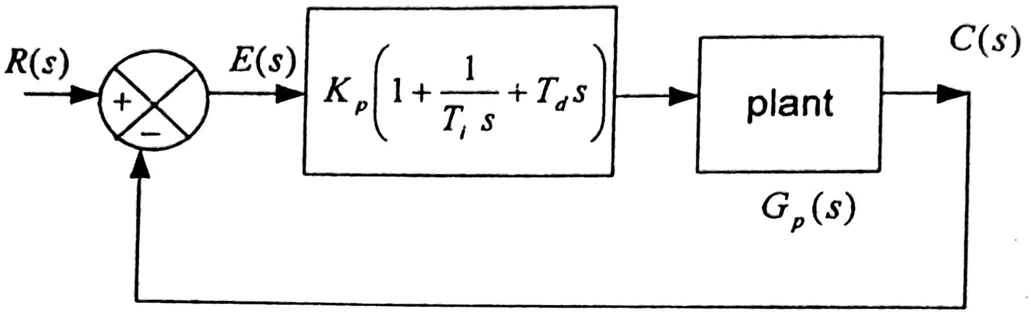
Table 2

Notice that the PID controller tuned by the second method of Ziegler–Nichols rules gives

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

$$= 0.6K_{cr}\left(1 + \frac{1}{0.5P_{cr}s} + 0.125P_{cr}s\right)$$

And by placing the designed PID controller given by transfer function $G_c(s)$ in the closed loop system as shown in figure below, the system performance [overshoot, rise time, settiling time and steady state error are improved (minimized)].



Stability in State Space

The system transfer function has $\det(s\mathbf{I} - \mathbf{A})$ in the denominator because of the presence of $(s\mathbf{I} - \mathbf{A})^{-1}$. Thus,

$$\det(s\mathbf{I} - \mathbf{A}) = 0$$

is the characteristic equation for the system from which the system poles can be found.

Example

Given the system

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 3 & 1 \\ 2 & 8 & 1 \\ -10 & -5 & -2 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\mathbf{x}$$

find out how many poles are in the left half-plane, in the right half-plane, and on the $j\omega$-axis.

$$(s\mathbf{I} - \mathbf{A}) = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 3 & 1 \\ 2 & 8 & 1 \\ -10 & -5 & -2 \end{bmatrix} = \begin{bmatrix} s & -3 & -1 \\ -2 & s-8 & -1 \\ 10 & 5 & s+2 \end{bmatrix}$$

Now find the $\det(s\mathbf{I} - \mathbf{A})$:

$$\det(s\mathbf{I} - \mathbf{A}) = s^3 - 6s^2 - 7s - 52$$

s^3	1	-7
s^2	$\cancel{-6}\ \ -3$	$\cancel{-52}\ \ -26$
s^1	$\cancel{-\frac{47}{3}}\ \ -1$	$\cancel{0}\ \ 0$
s^0	-26	

find out how many poles are in the left half-plane, in the right half-plane, and on the $j\omega$-axis.

Since there is one sign change in the first column, the system has one right-half plane pole and two left-half-plane poles. It is therefore unstable